

Non-symbolic and Symbolic Number Systems and Their Relationship to Mental Arithmetic among Female Undergraduates in Saudi Arabia

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Abstract

The study aimed to examine the relationship between non-symbolic and symbolic number systems with mental arithmetic in a sample of female undergraduates in Saudi Arabia. Seventy-six female undergraduates were recruited via the convenient sampling method and were asked to complete a paper-based mental arithmetic task to measure their mental arithmetic abilities. They were also asked to complete two computer-based tasks, dot comparison and dot estimation, to assess accuracy of the non-symbolic number system and symbolic number system, respectively. In addition, they were asked to complete an IQ test. Multiple regression analysis showed that only dot estimation predicted mental arithmetic. Nevertheless, mediation analysis showed that both IQ and dot comparison accuracy indirectly predicted mental arithmetic via the mediation of dot estimation. These findings indicated that the ability to precisely represent quantities in symbolic digits was a core ability that facilitated mental arithmetic in undergraduates. Based on these findings, the study recommended raising the awareness of the importance of the estimation ability in other math abilities among math teachers and course designers. Future research is also recommended to explore factors related to accuracy of the symbolic number system and the effectiveness of training number systems in improving mental arithmetic abilities.

Keywords: Number system, Mental arithmetic, Non-symbolic Representation, Symbolic representation, Female undergraduates.

أنظمة الأرقام الرمزية وغير الرمزية وعلاقتها بالحساب الذهني لدى طالبات البكالوريوس في المملكة العربية السعودية

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الملخص

هدفت الدراسة إلى اختبار العلاقة بين أنظمة الأرقام غير الرمزية والرمزية والحساب الذهني لدى عينة من طالبات البكالوريوس في المملكة العربية السعودية. جُمعت عينة عددها ٧٦ طالبة في مرحلة البكالوريوس بطريقة العينة المتسيرة، وطلب منهن إتمام اختبار ورقي للحساب الذهني لقياس قدراتهن في الحساب الذهني، وطلب منهن - أيضاً - إتمام مهمتين حاسوبيتين: مهمة مقارنة النقاط، وتقدير النقاط؛ لتقييم دقة نظام الأرقام غير الرمزية ونظام الأرقام الرمزية على التوالي، إضافة إلى ذلك، طلب منهن إتمام اختبار للذكاء. أظهرت نتائج الانحدار المتعدد أن تقدير النقاط - فقط - يتنبأ بالحساب الذهني، غير أن تحليل العامل الوسيط أظهر أن الذكاء والدقة في مقارنة النقاط تتنبأ بشكل غير مباشر بالحساب الذهني من خلال تقدير النقاط كعامل وسيط. هذه النتائج توضح أن القدرة على تمثيل الكميات من خلال الأرقام الرمزية بشكل دقيق هي قدرة جوهرية تيسر الحساب الذهني على طالبات البكالوريوس. بناءً على هذه النتائج، يُقترح رفع الوعي بأهمية قدرة التقدير في القدرات الرياضية الأخرى لدى معلمات الرياضيات ومصممي المقررات، ويمكن للدراسات المستقبلية أن تستكشف العوامل التي يمكن أن تتصل بدقة نظام الأرقام الرمزية، وفعالية تدريب نظام الأرقام في تحسين قدرات الحساب الذهني.

الكلمات المفتاحية: نظام الأعداد، الحساب الذهني، التمثيل غير الرمزي، التمثيل الرمزي، طالبات الجامعة.

Non-symbolic and Symbolic Number Systems and Their Relationship to Mental Arithmetic among Female Undergraduates in Saudi Arabia

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Introduction

The importance of math abilities is not restricted to math subjects. Undergraduates, even those who are specialized in humanities and education, need math to comprehend and deal with many concepts and facts, including but not limited to understanding ratios and percentages of an observed phenomenon, comparing durations and differences in progress, calculating averages, scoring scales and tests, and making informed decisions (Lamas et al., 2012). Math abilities consist of a wide range of abilities (Dehaen, 1992). One basic ability in math is the general sense of quantities, whereby individuals approximately estimate amounts of items or people in a certain context. This ability is an internal representation and processing system, called an approximate number system (Dehaen, 1992; De Smedt et al., 2013).

This approximate number system is integrated with a symbolic number system during math development and education (Zaleznik & Park, 2021). This integration between the two systems allow individuals to accurately represent and manipulate quantities and numbers (Jang & Cho, 2018; Liu et al., 2018). Failure to comprehend numerical concepts among adults is related to problems beyond low academic achievement. For instance, low numerical abilities are related to low self-efficacy (Lawson et al., 2007). This leads to questioning the nature of numerical abilities and how they relate to other math abilities.

The relationship between approximate number system and mental arithmetic, which consists of the basic calculations such as mental addition

and subtraction, is well documented for school students (Gilmore et al., 2014; Halberda & Feigenson, 2008; Inglis et al., 2011; Wang et al., 2020). However, in what way such relation works in university students has just been recently investigated (e.g., Guillaume et al., 2018; Norris et al., 2015), and results were contradicted (e.g., Atagi et al., 2016; Jang & Cho, 2018). It was found that the approximate number system is correlated with basic formal math tasks, including counting and simple calculation, but not complex ones, including fractions (Atagi et al., 2016). In contrast, Jang and Cho (2018) showed that non-symbolic approximate number system correlated with math reasoning but not with mental arithmetic in adults.

Furthermore, the relation between approximate number system and math abilities was found to be mediated by the accuracy of symbolic representation of quantities, including the ability to order symbolic digits (Lyons and Beilock, 2011) and comparing quantities with symbolic digits (Jang & Cho, 2018). These findings might reconcile the discrepancy between the appearance or absence of the relationship between approximate number system and mental arithmetic. However, the previous studies were applied on western countries (e.g., Lyons & Beilock, 2011) or eastern countries (Jang & Cho, 2018). To the best of the researcher knowledge, no similar studies has been performed in Saudi Arabia or Arab countries.

Hence, the current study aims to examine the relationships between the two number-systems, symbolic and non-symbolic, and mental arithmetic. It focuses only on females, as differences between males and females in math abilities are reported in several studies (e.g., Stoet & Geary, 2013; Zhu, 2007), showing advantage to males on females (Stoet & Geary, 2013). Understanding the cognitive factors related to math abilities can inform the best practice to enhance math abilities.

Therefore, the problem of the study is illustrated in the following question: To what extent mental arithmetic is predicted by the accuracy of the number system and the general IQ, and does the accuracy of the symbolic number system mediate the relationship between the innate non-symbolic number system and IQ and mental arithmetic in female undergraduates?

Questions of the study

The study aims to answer the following questions:

- 1- To what extent IQ, symbolic and non-symbolic number systems predict mental arithmetic in female undergraduates?
- 2- Does symbolic number system mediate the relation between IQ and mental arithmetic in female undergraduates?
- 3- Does symbolic number system mediate the relation between non-symbolic approximate number system and mental arithmetic in female undergraduates?

Aims of the study

The study aims to achieve the following:

- 1- To examine the predictability of both IQ, symbolic and non-symbolic number systems of mental arithmetic in female undergraduates?
- 2- To examine whether symbolic number system mediates the relationship between each of non-symbolic approximate number system and IQ, from one end, and mental arithmetic, from the other end in female undergraduates.

Significance of the study

Theoretical significance

- 1- The findings of the current study will enrich the field of numerical cognition by highlighting variables related to individual differences in math abilities in university students in Saudi Arabia.
- 2- The study will open a new path for researchers in Saudi and other Arabic countries to explore the accuracy of number systems and their relationship with other math abilities or other general cognitive abilities in different ages.
- 3- The study will provide researchers with measures of number systems and mental arithmetic, which have adequate psychometric properties.
- 4- The findings might encourage researchers to examine the effectiveness of training programs, based on tasks triggering the symbolic and non-symbolic approximate number systems, in improving mental arithmetic.

Applied significance

- 1- The study might raise undergraduates' awareness of variables that can be related to their math abilities.
- 2- The findings of the study can guide module designers in education to focus on several abilities that are related to improving formal high math abilities.
- 3- The study results might be used as guidelines to build tests for math competence in undergraduates, which can assess students' potentials for specific courses or programs.

Limits of the study

Temporal limits. The study was performed during the academic terms in 2018.

Spatial limits. The study was performed in King Saud University in Riyadh.

Objective limits. The study variables were: (a) Non-symbolic approximate number system, which was assessed with a computer-based dot comparison task, (b) Symbolic number system, which was assessed with a computer-based dot estimation task, (c) Mental arithmetic, assessed with a paper-based mental arithmetic task, and (d) IQ test, which was assessed with the brief version of Binet Intelligence Scale-fifth edition.

Definitions of terms and concepts

Approximate number system

Piazza (2010) introduced the following definition, 'Approximate number system is an intuition on approximate number quantities and their relations' (p.551).

The current study used a dot comparison computer-based task to assess non-symbolic approximate number system. The precision of the non-symbolic approximate number system was measured by accuracy, reaction time and Weber's fraction score, which is calculated with a formula built by Halberda and Feigenson (2008) to delineate the ratio to which one is sensitive to differences in quantities.

Symbolic number system

Symbolic number system is defined as, "The ability to label numbers with Arabic digits, which can symbolize and discretize any continuous

quantity' (Dehaene, 1997, p.5).

In the current study the symbolic number system was measured by a dot estimation computer-based task. The score for this task was obtained by calculating the percentage of errors in participants' responses, after excluding a set of trials which were used as a control for attention lapse (Chesney et al., 2015).

Mental arithmetic

Mental arithmetic is defined as 'mental processes used to comprehend numerals, perform mental calculations and produce an appropriate written or spoken numerical answer' (Dehaene, 1992, p.2).

It is also defined as an ability emerges from counting and includes the basic calculations: addition, subtraction, multiplication and division (Dehaen, 1992).

The current study used a timed paper and pencil test to measure mental arithmetic, and the participant's score in this task is the number of problems correctly solved in the given time.

Theoretical framework

Approximate number system is an innate ability that is shown in infants (Dehaene, 1992). It refers to one's ability to represent and process numerical magnitude information (Libertus, 2015). It includes two different systems: (a) non-symbolic approximate number system which refers to the ability to represent and process quantities and magnitudes without using digits (Dehaene, 1992; Halberda & Feijinson, 2008), and (b) symbolic number system which refers to the ability to precisely match numerals to quantities, using digits (Booth & Siegler, 2006).

Non-symbolic approximate number system accuracy increases with education (Piazza et al., 2013). This approximate number system enhances acquisition of formal math abilities and is independent from linguistic and verbal abilities. It is also related to quantities which are larger than four items. This is because subitizing is an automatic counting ability that takes over when the items are four or less (Hyde & Spelke, 2011).

A well-known task to measure the non-symbolic approximate number

system is the dot comparison task (Clayton et al., 2015; Halberda & Feigenson, 2008; Chesney et al., 2015; Prather, 2014). In this task two arrays of dots (blue and yellow) are presented and participants have to identify which group of dots are larger in quantity. The task should be carefully designed as the precision in dot comparison is sensitive to size of dots and areas of distribution of dots (Chesney et al., 2015) and time of presentation (Inglis & Gilmore, 2013). Variant visual cues in different protocols of the dot discrimination task might cause discrepant results between studies (Clayton et al., 2015; Gilmore et al., 2011; Smets et al., 2016).

Additionally, the dot comparison task is sensitive to both magnitude and distance between numbers (DeWind et al., 2015; Halberda & Feigenson, 2008). For instance, comparing five with eleven dots is easier than comparing five with nine dots. This is because the distance between the former numbers is larger than the distance between the latter numbers. Comparing eight with five dots is easier than comparing 58 with 55 dots. Although the distances in both comparisons are the same, the larger the numbers, the larger the noise in the non-symbolic approximate number system. This in turn slows reaction time and produces more errors.

There remains a comment on the measures used in the dot comparison task. Performance in this task is measured with reaction time, number of correct answers, and Weber's fraction. Measures of performance in task (e.g., accuracy, reaction time) can give different results (Krajcsi, 2020). The accuracy measure was found to be the most reliable measure for the dot comparison task (Inglis & Gilmore, 2014).

Although adults use abstract symbolic numbers to perform mental arithmetic, their processing of symbolic digits involve activation of non-symbolic quantities (Atagi et al., 2016; Dehaene, 1997). This symbolic number system can be measured by tasks, such as the dot estimation task, where a number of scattered plots is presented and the participant is instructed to give an estimation of the dots without counting (Chesney et al., 2015).

One account suggests that the non-symbolic approximate number system might be involved in facilitating the ability to match quantities to

symbolic digits. After the associations are built in early school years, the non-symbolic approximate number system is not related to formal math abilities (Halberda & Feigenson, 2008).

Another view suggests that specific brain regions in the parietal lobe, including intraparietal sulcus, are involved in both non-symbolic approximate number system and formal math tasks (Clark et al., 2017; Haist et al., 2014). Finally, some studies argued that confounded factors are related to significant findings of relations between math abilities and non-symbolic approximate number system (Chesney et al., 2015; Cleland & Bull, 2015), for instance, inhibition of size of dots and focusing on the quantities. Inhibition is indeed related to math abilities and was found to be a confounding factor in some dot comparison tasks, where size and ratios are not controlled to remove the inhibition effect (Clayton et al., 2015; DeWind et al., 2015; Gilmore et al., 2013; Mathews & Lewis, 2016). However, some tasks controlled this inhibitory effect and still showed a correlation between performance in the dot comparison task and math abilities (for a review see De Smedt, 2013).

All in all, a plausible account to investigate is the idea that non-symbolic approximate number system facilitates acquisition of symbolic numbers. This account can be integrated with the account that non-symbolic approximate number system is only correlated with simple math tasks (Halberda & Feigenson, 2008). One can assume that symbolic digits are represented and processed more efficiently when they activate non-symbolic approximate number system. This dual representation of numbers can improve performance in direct and simple math tasks, including mental arithmetic (Lyons & Beilock, 2011; Jang & Cho, 2018). Based on this, precision in matching quantities to numbers, measured by a dot estimation task, is assumed to correlate with mental arithmetic. In contrast non-symbolic approximation of quantities is assumed to be indirectly linked with mental arithmetic via the mediation of level of performance in the dot estimation task.

Literature review

1- Studies examining the relationship between non-symbolic approximate number system and math abilities.

Inglis et al. (2011). Aimed to examine the relation between non-symbolic approximate number system and a set of math abilities, including math achievement, conditioning inference and geometry. Sixty-four adults in the United Kingdom, who were recruited via a university pool, performed a computer-based dot comparison task. They also performed the Woodcock-Johnson 3 math achievement test, a paper-based test for conditional inference with 32 statements, and the Van Hiele geometry test. In addition, they were asked to complete the matrices subtest for analytic reasoning from WASI test in order to control for general cognitive abilities. The findings showed no correlations between non-symbolic approximate number system and any of the math tests, even after controlling for analytic reasoning. The researchers suggested that the relation between non-symbolic approximate number system and math abilities vanishes in adults.

A close but different approach was taken to examine the relation between non-symbolic approximate number system and mental arithmetic in undergraduates in different majors. Guillaume et al. (2013) tested 59 French undergraduates, half of them were from psychology department and the other half were from engineering department. The students completed a computer-based arithmetic task and the dot comparison task. The results showed significant differences between engineering and psychology students in mental arithmetic. However, the correlation between non-symbolic approximate number system and mental arithmetic was not significant. The researchers, therefore, argued that individual differences in mental arithmetic are not related to the non-symbolic approximate number system.

In 2015, Chesney et al. aimed to verify whether tasks assumed to measure non-symbolic approximate number system and symbolic number system are valid, and whether they are correlated with math abilities. They recruited 247 undergraduate students, males and females, from

Ohio State University in the United States to perform a set of computer-based tasks: subjective measure of numeric abilities (involving response to eight statements), objective measure of numeric abilities (including a set of problems), dot comparison task, dot ratio estimation, dot number estimation, non-symbolic number line, and symbolic number line. The parameters of the tasks were tested by Chesney et al. in a first study, which showed that controlling the size and distances between dots affected performance in the tasks. The results showed that all measures of approximate and symbolic number systems are correlated, including the dot comparison and dot estimation tasks, which will be used in the current study. Additionally, both tasks correlated with the subjective and objective measures of numeric abilities. The researchers suggest that these findings indicate that the tasks are related to a unified system, and that both types of non-symbolic and symbolic tasks should be considered when assessing the relationships between accuracy of number systems and numeric abilities.

Using Electroencephalography (EEG), Guillaume et al. (2018) aimed to examine the relation between approximate number system and EEG activities. 57 French adults performed mental arithmetic task at the beginning of the study. Then they performed the dot comparison task and a digit comparison task while EEG activities were recorded. The results show that EEG activities correlate with performance in the dot comparison task when the weber value (the value of the threshold of sensitivity to differences) is below 1.2, and that it is correlated with performance in the mental arithmetic task. Based on their findings, they argued that individual differences in non-symbolic approximate number system and its relation to mental arithmetic is evident when small ratios between compared dots are used.

2-Studies examining the mediatory role of symbolic number system between math abilities and number sense or general cognitive abilities.

Lyons & Beilock (2011) aimed to examine the relation between approximate number system, ordering numbers and complex math after controlling other non-numerical abilities, including letter comparison and ordering and working memory. Fifty-four undergraduates in the University

of Chicago in the United States completed a set of tasks: dot comparison, letter comparison, number ordering, letter ordering, four complex math tasks, and a working memory test. The results showed that participants' scores in the dot comparison task and number ordering task correlated with their scores in the complex math tasks. Additionally, number ordering mediated the relation between dot comparison and complex math. These correlations remain significant even after controlling the effects of working memory. Based on their findings, the researchers suggested that the relation between non-symbolic approximate number system and complex math is mediated by number ordering.

Jang and Cho (2018) aimed to study the relation between approximate number system and math abilities in children and adults. With regards, to adults, 53 undergraduates and postgraduates in Korea completed a set of tests: dot-dot comparison task, dot-number comparison task, arithmetic fluency test, math reasoning test, and Raven matrices for nonverbal reasoning. The results showed that scores in comparison between dots which represents non-symbolic approximate number system and dot-number comparison, which represents mapping quantities to numbers, correlate with math reasoning but do not correlate with math fluency. Additionally, mapping quantities to numbers mediates the relation between scores of dot comparison and math reasoning. They explained this finding by the idea that reasoning requires activating number magnitude when thinking about the best approach to solve the math problems with reasoning.

Comment on previous studies

The above studies focused on the relationship between precision of the number systems and math abilities in adult populations in western (e.g., (Guillaume et al., 2018) and eastern countries (Jang & Cho, 2018). Apparently, no similar studies were found in the Arabian countries.

At a very basic level, studies that focused on the relation between dot comparison, as a measure for approximate number system, and math abilities in adults had revealed contradicted findings. Some studies showed a positive correlation between performance in dot-comparison and undergraduates scores in subjective and objective measures of numeric

abilities (Chesney et al., 2015), mental arithmetic problems (Guillaume et al., 2018), math reasoning (Jang & Cho, 2018) or complex math tasks (Lyons & Beilock, 2012).

On the other hand, the correlation between scores of dot comparison and math abilities was absent in other studies on adults (Guillaume et al., 2013; Inglis et al., 2011). For instance, Inglis et al. (2011) and Price et al. (2012) failed to find such correlation between dot comparison and math achievement measured by the woodcock-Johnson test. Correlations were neither found between scores of dot comparison and geometry abilities nor with conditional inference (Inglis et al., 2011). Similarly, performance in dot comparison did not correlate with mental arithmetic (Guillaume et al., 2013; Jang & Cho, 2018).

Other studies examined the relationship between precision of number sense and math abilities after controlling for general cognitive abilities. Inglis et al. (2011) controlled for the analytic reasoning ability measured by the matrices subtest in WASI, but the correlation remained insignificant. For the same purpose, Jang and Cho (2018) used the Raven matrices test to control the effect of non-verbal reasoning. Lyons & Beilock (2012) controlled for working memory as a general cognitive ability, and the correlation remained significant. The effect of the undergraduate major was also tested. Guillaume et al., (2013) tested two groups of participants who were related to either of two different majors, psychology or engineering, and found that the major of the student compared to the precision of the approximate number system is a better predictor for performance in mental arithmetic.

The findings are complicated by the notion that studies drew attention to precision of symbolic number system, assessed by the dot estimation task, which is related to both non-symbolic number system and math abilities (Chesney et al., 2015). Lyons and Beilock (2012) showed that symbolic number system, assessed by performance of number ordering task, mediates the relationship between approximate number system and complex math abilities. Jang and Cho (2018) showed similar mediatory role of symbolic number system between approximate number system and math reasoning, but the task they used for symbolic number system was a

dot-number comparison.

The aim of this current research was to shed light on the nature of the relation between approximate number system and mental arithmetic abilities in undergraduates in Saudi Arabia. If accurate mapping between the symbolic numbers and their quantities is the determinant of the relation between approximate number system and mental arithmetic performance (Lyons & Beilock, 2011), then scores of the dot estimation task should show a mediatory role between dot comparison scores and mental arithmetic scores. In contrast, if the correlation between approximate number system and mental arithmetic performance is attributed to approximate sense of numbers (Guillaume et al., 2018), then the dot comparison task would show a direct correlation with mental arithmetic scores. The current study used the dot comparison task, which was consistently used in previous studies to assess the approximate number system and the dot estimation task for the symbolic number system (Chesney et al., 2015). For the math abilities, the focus was on mental arithmetic, as it is a basic math ability. Additionally, the current study aims to examine the mediatory role of symbolic number system, if found, in terms of whether it mediates the relationship between math abilities and the intuitive approximate number system or the general cognitive ability. Hence, it did not choose any of the nonverbal measures used in previous studies (e.g., Inglis et al., 2011; Jang & Cho, 2018), and used the brief battery of Stanford-Binet 5 Intelligence Scale as it is a brief measure of the general cognitive ability (Faraj, 2011).

Method and Procedures

Method

The study used the correlative method, which is useful in examining the direction and strength of correlation between variables. This method is suitable for the questions of the study, which aim to investigate the relationship between a number of variables (Vogt et al., 2017).

Participants

Seventy-six healthy female undergraduate students, mean age = 21.118; standard deviation = 1.681 years, were recruited for the study via the

convenient sampling method. Additional two participants were recruited but excluded because of technical failure in extracting their data from the computer-based tasks. The adequate number of participants was found to be 77. It was determined by G*Power Software, whereby the power of the statistics was 0.80, the error probability was 0.05 and the effect size was 0.15. All participants reported no diagnosis of learning difficulty, and they were studying in the humanities colleges, either psychology students or taking a course in psychology. As participants were not studying math courses during the time of the study, no measure of math achievement is provided.

Instruments

Dot comparison. The dot comparison task is used to measure the precision of non-symbolic approximate number system by differentiating two quantities without the use of symbols (Chesney et al., 2015). This task is based on the task designed by Halberda and Feigenson (2008). The task is to judge which of two sets (yellow dots & blue dots) has larger numbers of dots. It consists of 56 slides, and four practice slides. The yellow and blue dots are presented either in the right or left half of a grey background, and their locations are switched across slides. The number of dots in each set vary between 5 and 22, and the ratio between the two sets, in each trial, is one of seven ratios: '0.8, 0.11, 0.15, 0.18, 1.25, 1.5, 2'.

Several measures are used to assess performance in the dot comparison task, including accuracy, reaction time and weber fraction (w) (Seigler, 2018). The Weber fraction is calculated based on the method of Halberda and Feigenson (2008).

Validity and reliability. The dot comparison task had several versions, and several studies in western countries showed its good level of validity and reliability (e. g., Chesney et al., 2015). The current study tested its validity by content validity, where the task was reviewed by three judges who are specialized in psychology. The judges confirmed the accuracy of the task parameters and the clarity of the task and suitability to its purpose. However, they spotted mismatch between the task description and a couple of slides, and those were corrected. Additionally, internal consistency of

the task was examined on the first 30 participants in this study. Pearson correlations were calculated between the score of each ratio and the total score, and all correlations were above 0.3. See Table 1 for correlations between the scores of each ratio and the total score. Reliability was also calculated using split half reliability. Brown-Spearman correlation was 0.692.

Table (1)
Correlations between the scores of performance of 30
participants in each ratio and their total scores

Ratios	R	P
Ratio: 0.8	0.71	0.001
Ratio: 0.11	0.71	0.001
Ratio: 0.15	0.631	0.001
Ratio: 0.18	0.705	0.001
Ratio: 1.25	0.636	0.001
Ratio: 1.5	0.448	0.01
Ratio: 2.0	0.39	0.05

Dot estimation. The dot estimation task involves participants in estimating the number of white dots presented on a grey screen. Accuracy in estimation reflects precision of the symbolic number system and matching digits with representative quantities (Chesney et al., 2015). The sizes of dots varies (diameters ranges between 1.5-2.5cm). The task includes 32 slides, in addition to four practice slides. The number of dots in each of the 32 trials ranges between 5 and 18.

Scores are obtained by calculating the absolute differences between estimations and exact numbers of the items with dots above eight dots, then dividing the scores by the exact numbers and multiplying the total by 100 for each item. Items with eight dots or less are used to ensure that there was no attention lapse, as undergraduates were supposed to be 100% accurate in these items (Chesney et al., 2015).

Validity and reliability. The task was used in previous studies with different numbers of items, sizes and locations of dots and showed good

levels of validity and reliability (e. g., Chesney et al., 2015). To assess the validity of the task with its current parameters and in the current population, content validity and internal consistency were used. The same three judges, who judged the validity of dot discrimination task, reviewed the dot estimation task according to its accuracy, clarity and suitability for measuring the accuracy of symbolic system, and they reported that it is valid, with only a suggestion to increase the scale of the grey background to increase the contrast. This amendment was performed. Similar to the dot comparison task, internal consistency was calculated on the data of the first thirty participants in the study. The correlations were between, $r = 0.867$ and $r = 0.779$, $p < 0.001$. The reliability was also calculated using split half reliability, and Brown-Spearman was 0.6.

Mental arithmetic. Twenty arithmetic problems (10 addition & 10 subtraction) were presented in A4 paper, and the time was limited to 3 minutes. Each problem had two digit operants. This task measures fluency of mental arithmetic and is based on a subtest in Woodcock-Johnson test, used in previous studies (Inglis et al., 2011; Price et al., 2012).

Validity and reliability. The task was designed by the researcher and was judged by the same three judges in terms of the consistency of level of difficulty for all problems, and no amendments were suggested. Hence, the score of each problem was either 0 or 1, internal consistency was calculated using the point biserial correlation between scores of each problem in the test and the total scores. See Table 2 for the correlation values and levels of significance. Kuder-Richardson reliability was 0.851, which indicates a good level of reliability.

Table (2)
Correlations between the scores of performance in each problem
and the total scores in the mental arithmetic test

Ratios	Point biserial correlation	P
Problem 1	0.281	0.01
Problem 2	0.226	0.05
Problem 3	0.464	0.001
Problem 4	0.432	0.001

Table (2)

Ratios	Point biserial correlation	P
Problem 5	0.541	0.001
Problem 6	0.31	0.01
Problem 7	0.327	0.01
Problem 8	0.536	0.001
Problem 9	0.454	0.001
Problem 10	0.498	0.001
Problem 11	0.695	0.001
Problem 12	0.637	0.001
Problem 13	0.516	0.001
Problem 14	0.638	0.001
Problem 15	0.641	0.001
Problem 16	0.492	0.001
Problem 17	0.624	0.001
Problem 18	0.659	0.001
Problem 19	0.685	0.001
Problem 20	0.399	0.001

The Brief battery of Stanford-Binet 5 intelligence scale. The Arabic version of the Stanford-Binet 5 intelligence scale standardized by Faraj (2011) was used to assess IQ. It is an individually administered test of intelligence. Administration of the brief battery includes verbal knowledge and non-verbal fluid intelligence which are the routing subtests for the full test. The scores of the two routing subtests can be used to estimate the general intelligence score. This general intelligence score is a standard age score with a mean of 100 and a standard deviation of 15.

Procedures

The study started by designing and judging the instruments. Then an approval from the research ethics committee in King Saud University was obtained. A preliminary study was performed on 30 participants, and since

no amendments were performed, the participants from the preliminary study were included in the main sample.

Participants were tested individually in a testing cubicle in the psychology department. At the beginning of the study, the participant signed a consent form prior to the study. Each session lasted approximately 45 min. The order of the tasks was counterbalanced across participants.

For the two computer-based tasks, the dot estimation and dot discrimination, the participants were seated in a comfortable distance from the laptop. The researcher explained the practice trials and allowed the participant to perform the four practice trials. All participants were able to perform the practice trials with no further assistant. Following that the researcher entered the participant's number and started the main tasks.

For the dot comparison task, participants were instructed to judge which of two sets (yellow dots & blue dots) has larger numbers of dots. The trials were presented on 15 inches laptop screen with PsychoPy software (Peirce, 2007), and each trial was presented for 2000ms or until the participant responded. Participants responded by pressing either the 'right' or 'left' arrow keys, and feedback, either correct or incorrect, was shown on the screen.

For the dot estimation task, participants viewed slides presented on 15 inches laptop screen and controlled via PsychoPy software (Peirce, 2007). Each slide was presented for 2000ms or until the participant gave her verbal response. The order of slides was randomized across participants, and the researcher wrote the participants' answers.

In the mental arithmetic task, the researcher handled the test and a pencil to the participant and informed the participant that she would be asked to solve 20 addition and subtraction problems within three minutes. They were supposed to mentally solve the problems without scratching medium answers on the paper.

Then the researcher asked the participant to turn the page and started the stopwatch. In the IQ test, the researcher sat on the opposite side of the table in front of the participant and visually presented the questions to the participant (the words in the verbal knowledge test and matrices in the nonverbal analytic reasoning test). The cover of the booklet of the

questions was used as a stand for the booklet so that participants can see the questions clearly. The researcher informed the participants in the verbal knowledge test that their task is to give a complete definition of each presented word. In the analytic reasoning test, the researcher informed the participants that they were asked to complete each matrix from one alternative choices presented in the bottom of each page. The participants gave their responses verbally and the researcher recorded their answers. In some occasions where the participant answer required clarification as stated by the test guide, the researcher asked for more explanation. Participants were debriefed and given their results after the study.

Method of analysis

Pearson correlation and multiple linear regression were used to answer the first question of the study, and mediation analysis with the mediation package with bootstrapping method (Preacher & Hayes, 2004) was used for the second and third questions.

Results

Measures were scored according to the method explained in the instruments section. See Table 3 for means and standard deviations in each task.

Table (3)
Means and standard deviations of the study variables

Variables	M	SD
IQ	108.539	3.403
Percentage of errors in dot estimation	14.068	7.307
Mental arithmetic	12.618	4.326
Dot comparison (accuracy)	86.677	7.379
Dot comparison (RT)	1.343	0.465
Weber's Fraction	0.110	0.076

In relation to the first question: To what extent IQ, symbolic and non-symbolic number systems predict mental arithmetic in undergraduate students?

Correlations between percentage of errors in dot estimation (%EDE) and mental arithmetic and between them and each of IQ and dot comparison accuracy (DCA) are shown in Table 4. As the correlations were carried out only as a preliminary investigation, alpha values for correlations were corrected via false discovery rate method (FDR) by Benjamini and Hochberg (1995). Only accuracy measure of the dot comparison task was included in further analysis as the two other measures did not show any significant correlations with other variables.

Table (4)
Correlations between IQ, mental arithmetic and percentage of errors in dot estimation

Variables	IQ		DCA		Mental arithmetic	
	r	p	R	P	r	P
%EDE	-0.279	0.007	-0.237	0.020	-0.344	0.001
Mental arithmetic	0.242	0.017	0.052	0.328		

As shown in Table 5, the first multiple regression inserted IQ, percentage of errors in dot estimation and dot comparison accuracy as predictors and mental arithmetic as an outcome. The results showed that only percentage of errors in dot estimation predicted mental arithmetic, $t = -2.601$, $p = 0.011$, $F(72) = 3.967$, $p = 0.011$. For the second multiple regression, where IQ and dot comparison accuracy were predictors, and percentage of errors in dot estimation was an outcome, both IQ and dot comparison accuracy predicted percentage of errors in dot estimation, $t = -2.154$, $p = 0.035$, and $t = -2.544$, $p = 0.013$, respectively, $F(73) = 5.595$, $p = 0.005$.

Table (5)
Values of two multiple regression analysis

Outcome	Predictors	r^2	F	P	B	t	P
Mental arithmetic	IQ	0.106	3.967	0.011	0.157	1.382	0.171
	%EDE				-0.305	-2.601	0.011
	DCA				-0.021	-0.190	0.850
%EDE	IQ	0.109	5.595	0.005	-0.235	-2.154	0.035
	DCA				-0.277	-2.544	0.013

It seems from the above results that IQ and dot comparison accuracy predicted percentage of errors in dot estimation, and that only percentage of errors in dot estimation predicted mental arithmetic.

In relation to the second and third questions: Do symbolic number system mediate the relation between each of IQ and approximate number system from one side and mental arithmetic?

It was of value to examine the mediatory effect of dot estimation between each of IQ and dot comparison accuracy from one side and mental arithmetic from another side. Therefore, mediation analysis calculated averaged causal mediation effect (ACME) via nonparametric procedures. The mediation package with bootstrapping method (Preacher & Hayes, 2004) was used in R studio for the mediation analysis.

For the mediation model, where IQ was a predictor, percentage of errors in dot estimation a mediator, and mental arithmetic an outcome, ACME was significant, 0.25, $p = 0.028$. However, no significant direct relation was found between IQ and mental arithmetic, 0.47, $p = 0.166$.

With regards to the second mediation model, the model included dot comparison accuracy as a predictor, percentage of errors in dot estimation as a mediator, and mental arithmetic as an outcome. The results showed a significant averaged causal mediation effect ACME, 0.10, $p = 0.03$, while no significant direct relation between dot comparison accuracy and mental arithmetic, 10, $p = 0.74$. See Figure 1 for the mediatory effect of dot estimation.

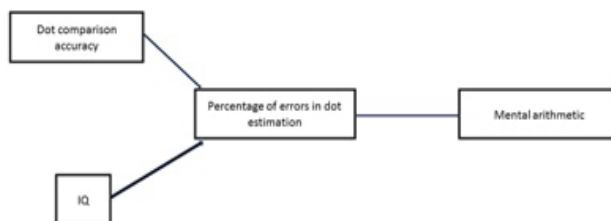


Figure 1

IQ and dot comparison independently and indirectly predict mental arithmetic via the mediation of dot estimation

Discussion

The current study aimed to examine the relations between IQ, non-symbolic and symbolic number systems and mental arithmetic in undergraduate students. The results of the study showed that only symbolic number system predicts mental arithmetic. The results also showed that IQ and non-symbolic approximate number system predict mental arithmetic via the mediation of symbolic number system. These findings are thoroughly discussed below.

Discussion of the results of the first question, related to the predictability of both approximate number systems, symbolic and non-symbolic, and IQ of mental arithmetic

Only symbolic number system predicted mental arithmetic. This is consistent with the prediction that non-symbolic number system facilitates the development of the symbolic number system, and symbolic number system, in turn, becomes the system in use for mental calculations (Dehaene, 1997). This finding is consistent with the findings of (Inglis et al., 2011; Szucs & Myers, 2017), who showed no significant relation between non-symbolic approximate number system and math abilities. Nevertheless, the finding are at odds with several other findings in the literature. For instance, DeWing and Brannon (2012) showed a strong correlation between precision of non-symbolic approximate number system and math achievement. However, in addition to the differences between the contexts of the studies, it should be noted that the tasks used here is a timed mental arithmetic task, while in their study, the scores of math were the score of SAT which combines multiple math abilities, including geometry and reasoning. Additionally, the time of presentation of dots in the dot comparison task was limited to 200ms in DeWing's and Brannon's (2012) study. In contrast, they were presented for 2000ms in the current study. These differences might suggest that time pressure might be a key factor for precision of the approximate number system. Indeed, Cicchini et al. (2014) showed that task settings can largely change levels of participants precision in approximate number system.

The findings of the current study speculate that although precision of approximate number system is related to mental arithmetic (DeWing &

Brannon, 2012), this relation might be influenced by several variables, including type of representation, symbolic or non-symbolic, and time pressure.

With regards to the relationship between IQ and mental arithmetic, the current study did not show a significant result. Previous studies (e.g., Jang & Cho, 2018; Lyons & Beilock, 2011) controlled the role of general cognitive abilities, such as nonverbal reasoning, to verify whether mental arithmetic abilities are related to specific domain abilities (e.g., number systems) or general cognitive abilities, and showed no significant role for general cognitive abilities in controlling the relationship between approximate number system and math abilities. However, the method of analysis here in the current study is different, as IQ is entered in the regression model as a predictor, while in previous studies it was a covariate. This issue is inspected further in the second question.

Discussion of the results of the second and third questions, related to the mediatory role of the symbolic representation of numerals between each of IQ and non-symbolic number system with mental arithmetic

In relation to the first step of the mediation analysis whereby the predictability of IQ and non-symbolic number system of symbolic number system was tested, the results were significant for both predictors. The ability to precisely match quantities with symbolic digits is significantly predicted by IQ and precision of non-symbolic representation and processing of quantities. These findings are consistent with the findings of Chesney et al. (2015), which illustrate that dot comparison and dot number estimation tasks are correlated. This indicates that albeit non-symbolic and symbolic number systems are related to different tasks, they are partially integrated (Liu, Schunn, Fiez, & Libertus, 2018).

However, dot estimation can predict mental arithmetic performance beyond the role of dot comparison task. Data from an eye-tracking study showed that eye-fixation on non-symbolic dot comparison stimuli is larger than eye-fixation during symbolic number comparison, indicating that speed of processing is facilitated by using symbolic numbers (Price et al., 2017).

The mediatory role of symbolic number system between non-symbolic number system and mental arithmetic was significant. This finding is consistent with the findings of Lyons and Beilock (2011), who showed that ordinal representation of numbers mediates the relation between non-symbolic approximate number system and math abilities.

This mediatory role for symbolic number system between the non-symbolic system and math abilities reconciles the discrepant findings of several studies, who showed no correlation between non-symbolic approximate number system and math abilities in adults (e.g., Inglis and colleagues, 2011) and their counterparts who showed significant relations between the approximate number system and math abilities (e.g., Clark et al., 2017; DeWing & Brannon, 2012). These findings illustrate the importance of differentiating between symbolic and non-symbolic number systems when studying the relation between approximate number system and math abilities. Additionally, the findings indicate the importance of the ability to match symbolic digits with quantities in a fast and precise mental processing. Indeed, recent electrophysiological studies (e.g., Zaleznk & Park, 2021) started to explore brain regions responsible for symbolic digit representations. Zaleznk and Park (2021) argue that this advancement in studying the number sense in adults might lead to better practices in assessing cognitive abilities related to number sense precision.

Finally, with regards to the mediatory role of symbolic approximate number system between IQ and mental arithmetic, the results showed a significant mediation. This finding indicates that math abilities are related to both domain specific and general cognitive abilities. Indeed, the finding that mental arithmetic is not only related to domain specific cognitive abilities is supported by findings from previous studies. Ataji et al. (2016) found that presenting visual cues with symbolic fractions facilitates performance in a comparison task. Visual cues might be useful in mapping symbols with their magnitudes. This might be also investigated in future studies by examining the role of visual-spatial working memory in solving both mental arithmetic and magnitude discrimination tasks.

Conclusion

The role of non-symbolic approximate number system is not attenuated in female undergraduates in Saudi Arabia but is mediated by accuracy of symbolic representation and processing of numbers. Symbolic representation of numbers also mediates the relationship between IQ and mental arithmetic. Hence, studying the factors related to the precision of the symbolic number system is a track towards understanding individual differences in math abilities. These findings can have implications on how adults who are incompetent in arithmetic can be trained on matching non-symbolic and symbolic representations of quantities.

Recommendations and suggestions

- 1- Colleges and universities are required to test the accuracy of the number sense in the entrance exams as this ability is a core ability for other academic abilities.
- 2- Decision makers are required to direct educators to pay attention to the importance of non-symbolic and symbolic number systems in mathematics teaching and evaluation.
- 3- Further studies are required to examine the generalizability of the current findings in other groups in the Saudi society and to include males and females.
- 4- Further studies are required to compare males and females' accuracy in number systems and math abilities in different tracks of studies in order to understand whether females' inclination not to specialize in math and sciences is partially influenced by the accuracy of their number systems.
- 5- The role of visuospatial working memory and other general cognitive abilities in both non-symbolic and symbolic number system and mental arithmetic can be examined in future studies.
- 6- Applied studies might test the effectiveness of training on approximate number system in enhancing math abilities.

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