Investigation of grade 11 Lebanese Students' Misconceptions about the Concept of First Derivative of a Function Defined Graphically

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ABSTRACT

This research attempts to investigate the misconceptions, and their eventual causes, of the students in the concept of the first derivative of a function defined graphically. A group of 90 grade 11 participants was chosen from two public secondary schools in the southern suburb of Beirut, Lebanon. A test was administered to the participants; this test contains two parts: I) the graphs of functions are given and derivatives are to be found; II) the derivative(s) of a function at a point and other calculus concepts closely related to the subject are given and convenient graphs are to be drawn. The results of the test were categorized depending on the misconceptions of the participants, and then 36 participants were chosen to have one-to-one interviews. Their 4 teachers were also interviewed. The misconceptions of the participants vary among eight forms and the reasons of these misconceptions vary among eight eventual causes.

خلاصة

تهدف هذه الدراسة الى اظهار التصورات الخاطئة، وأسبابها المحتملة، عند التلاميذ بما يخص مفهوم المشتقة من الدالة المعرّفة بالرسم البياني. لهذا الهدف تم اختيار مجموعة من 90 تلميذًا، وأساتذتهم

الاربعة، من الصف الثاني الثانوي من مدرستين رسميتين من الضاحية الجنوبية لبيروت. خضع جميع التلاميذ لاختبار خطي مكوّن من جزأين: 1) الرسم البياني للدالة معطى ويطلب معلومات عن المشتقة؛ 2) يطلب رسم بياني للدالة انطلاقا من معطيات حول الدالة ومشتقتها. نتائج الاختبار بوّبت بحسب التصورات الخاطئة. بعدها، تم اختيار 36 تلميذًا من المجموعة لإجراء مقابلات فردية معهم ومع أساتذتهم. أخيرًا، تم استخراج 8 تصورات خاطئة و8 أسباب محتملة لها.

1 Introduction

The concept of the derivative of a function is used, in addition to mathematics, in many subject matters in both schools and universities. In Lebanon, pre-tertiary grades 11 and 12 students learn the derivative of a function in physics, chemistry, biology, economics, and mathematics. This research sheds lights on the importance of the graphical context of the first derivative of a function in the Lebanese curriculum, so that teachers give more efforts to achieve this concept in both their evaluation and teaching strategies. The research intends to help pre-service and in-service teachers to improve their strategies and ways of teaching, and to give them wider points of views about the misconceptions of the concept of the first derivative of a function. Indeed, teachers usually accumulate the misconceptions of any concept through their teaching experience (Lauten, Graham, & Ferrini-Mundy, 1994). This research helps also the trainers and teachers, in the faculty of education, see and show (to their students) the misconceptions in the concept of the derivative of a function from a graphical perspective, then searching for strategies to treat it even to prevent it.

The purposes for the research are to investigate the students' misconceptions and their eventual causes in the concept of first derivative of a function defined graphically. Consistent with the purposes of the research, we focused on two main research questions:

1. What are the students' misconceptions in the concept of first derivative of a function defined graphically?
2. What are the eventual causes of the students' misconceptions in the concept of first derivative of a function defined graphically?

2 Literature Review

2.1.1 Theoretical background

The term "misconception" does not have a unilateral definition since various authors approached it from diverse viewpoints. Hammer (1996) thought that students' misconceptions (a) are strongly held and stable cognitive structures; (b) differ from expert understanding; (c) affect in a fundamental sense how students understand natural phenomena and scientific explanations; and (d) must be overcome, avoided, or eliminated for students to achieve expert understanding. Chi and Roscoe (2002) have a consistent view about misconception with Hammer. They referred to "preconception" as naïve knowledge that could be easily revised and removed, while "misconception" was naïve knowledge that was strong to be change. Santi and Sbaragli (2008) defined a "conception" and a "misconception". They stated that "the student acquires a correct model of the concept when he masters the coordination of a set of representations, relative to that concept, that is stable and effective in facing diverse mathematical situations" while a misconception is "a set of representations that worked well in previous situations but it is inappropriate in a new one." D'Amore and Sbaragli (2005) stated that "misconceptions are considered as steps the students must go through, that must be controlled under a didactic point of view and that are not an obstacle for students' future learning if they are bound to weak and unstable images of the concept; they represent, instead, an obstacle to learning if they are rooted in strong and stable models." D'Amore (1999) defined a misconception as "a wrong concept and therefore it is an event to avoid; but it must not be seen as a totally and certainly negative situation: we cannot exclude that to reach the construction of a concept, it is necessary to go through a temporary misconception that is being arranged." In this research, the term "misconception" will be defined using the definition of D'Amore (1999) and the statement of D'Amore and Sbaragli (2005) where "misconceptions are considered as steps the students must go through and that must be controlled under a didactic point of view".

2.2 Literature Review

Many researchers examined the difficulty of differentiation and found that students think about differentiation as a difficult topic in mathematics, and that students do not really understand the concepts and procedures in differentiation. They found that students have a predominant reliance on the need for algebraic formulas when dealing with the function concept and that this predominant reliance may contribute to the development of a restricted notion, or concept image, of function. They also found that students struggle with graphing the derivative of a function without having its algebraic representation. In addition, they found that students' graphing skills are weak when cognitively combining, simultaneously, interval information of the derivative and critical points. (Orton, 1983; White & Mitchelmore, 1996; Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Eisenberg, 1992; Schwingendorf, Hawks, & Beineke, 1992; Asiala, Cottrill, Dubinsky, & Schwingendorf, 1997; Stahley, 2001; Leinhardt, Zaslavsky, & Stein, 1990; Lauten *et al.*, 1994; Baker, Cooley, & Trigueros, 2000; Ferrini-Mundy & Graham, 1994; Stahley, 2001).

Selden, Selden, and Mason (1994) found that students have difficulties in connecting functions and their graphical representations because of the traditional instructional methods that tend to emphasize the construction of graphs from algebraic formulas or tables, but do little in the way of the reverse, from graphs to algebraic formulas or tables. They stated that even students who performed very well on routine calculus problems found great difficulty and had little or no success in dealing with problems that were non-routine. Ubuz (2001) found that students seem to think about different concepts as one concept. The reasons appeared to be: "(a) the lack of discrimination of concepts which occur in the same context or the confusion of a concept with another concept describing a different feature of the same situation; (b) the inappropriate extension of a specific case to a general case; and (c) the lack of understanding of graphical representation." Orton (1983) showed that students have little intuitive understanding as well as some fundamental misconceptions about the concept of derivative. The routine aspects of differentiation could be performed quite well. However, when the students were presented with a function they had not seen before, the frequency of errors increased indicating strong reliance on algorithmic steps without a conceptual understanding. Orton (1983) also concluded that other areas of student difficulty are related to the ideas of a rate of change at a point versus the average rate of change over an interval. In addition, he concluded that students confuse the derivative of a function at a point and the value of the function at that point, or the *y*-value of the point of tangency. Amit and Vinner (1990) and Asiala *et al.* (1997) found many students' common misconceptions about the graphical interpretation of the derivative of a function. They concluded that some students equate the derivative of a function with the equation for the line tangent to the graph of the function at a given point. Ubuz (2001) reported that students' common misconceptions on derivative were as follows: "(a) derivative at a point gives the function of a derivative; (b) tangent equation is the derivative function; (c) derivative at a point is the tangent equation; and (d) derivative at a point is the value of the tangent equation at that point."

Zandieh (2000), Hähkiöniemi (2006), and Roorda (2010) distinguished among four levels of the graphical representation of the derivative of a function. These levels are graph, average slope, slope of a tangent, and graph of derivative. Heid (1988) noted that "the notion of derivative as slope or rate of change … fades quickly with disuse because students learn to rely on memorized procedures for a small number of exercise types". Orton (1983) found that the rule where one divides the difference in *y* by the difference in *x* to obtain a rate was not elementary for a large number of students because the ideas related to the rate of change are basically concerned with ratio and proportion. Leinhardt *et al.* (1990) argued that students have difficulty in computing slopes from graphs. They call for a greater emphasis on the reverse, from graphs to algebraic formulas or tables in an effort to help students deal with difficulties they encounter with graphical problems. Schoenfeld, Smith, and Arcavi (1990) made a long term study of student's construction of the concept of slope. They found that what appears to most people working at any level in mathematics to be a very simple concept can present serious difficulties, even to students that are considerably strong.

3 Methodology

3.1.1 Participants

The participants for the research are 90 grade-eleven students and their 4 teachers. Each one of the four teachers has a bachelor degree in mathematics, and two out of them have a teaching diploma degree in teaching mathematics for secondary classes. The 90 students were chosen randomly from two randomly selected public secondary schools in the Southern Suburb of Beirut. In the first school, there are four grade 11 (Sciences) sections having four different mathematics teachers, out of which two sections were selected randomly containing 54 students. In the second school, there are three grade 11 (Sciences) sections having three different mathematics teachers, out of which two sections were selected randomly containing 36 students.

3.2 Tools

In this research, there are two tools that are used: Test and one-to-one interview. The reason of choosing both test and interviews was to strengthen the validity and reliability of the data. Indeed, interpreting the learning behavior of students in calculus needs examining the students' written work and analyzing information via interviews (Strauss & Juliet, 1990).

Interviews were conducted because tests have a disadvantage in that they cannot, sometimes, probe deeply into participants' proper understanding of the concept; once the questions are set, they remain as they are. All interview questions emerged from the test responses and were used in investigating deeper graphical understanding of the first derivative of a function. A major objective is to detect if the error committed results from a lack of attention or from a misconception which is verbalized.

The test is a 50-minutes test that contains 12 independent questions was prepared and applied on 90 students (See Appendix A). The test was organized in a way that it contains three main parts: part *nulla*: It contains Question 1 that asks participants to set up the table of variations of a function that is defined graphically; the question was set to warm up the participants in refreshing their memories about the subject and has nothing to do with the data for the research. Part I: From Question 2 to Question 8, where the graph of a function is given and the participant has to answer a question related to the number derivative of that function; in these questions, participants are given graphs and are asked to find derivatives with justification. Part II: From Question 9 to Question 12, where participants are asked to draw a possible graph of a function that satisfies one or more given conditions that are related to the derivative of the function at a point and to other closely related calculus concepts; in these questions, participants are given the number derivative and are asked to draw possible graphs. The test was validated and piloted before it was applied on the 90 participants[20].

There are two sets of one-to-one interviews: One set with 36 selected students from the 90 participants and another set with their 4 teachers. Interviews allow for an in-depth discussion that could potentially reveal information not otherwise found through surveys or other quantitative forms of data collection (Mertler, 2009). There are two goals for the one-to-one interviews with the students: First to check whether the responses of the participants really reflect their level of understanding[21] and second to clarify unclear responses by the participants to tell whether they have any misconception about any point in the subject (Some participants wrote correct responses that are not justified while others wrote correct responses that are justified briefly or incompletely). After correcting, classifying, and

[20] For more information, read the Master thesis of Ballout, H. (2014)

[21] This research is extracted from a Master research in Mathematics Education (Ballout, 2014)). One of its purposes is seeking the understanding levels of the students based on the APOS theory.

categorizing the students' misconceptions in the test, the researchers selected the students that would be interviewed, depending on their misconceptions and unclear responses in the test. In each misconception, 25% of the participants were selected and interviewed because one of the main purposes of the research is to investigate the misconceptions of the students in the concept and, through the interview, one may go deeply into the interviewee' proper understanding of the concept; 20% of the participants with unclear responses were selected. As a total, 36 one-to-one interviews were conducted with 36 participants. Some of these participants were interviewed in the two parts, the misconception and the unclear responses. Concerning the 4 interviews with the four teachers who taught the 90 students in the test, there are two goals of interviewing the teachers: First to check whether the teacher is aware of his/her students' misconceptions about the concept or not and second to check the material each teacher uses in his/her class, whether s/he sticks to the assigned textbook or uses outside materials[22].

3.3 Research Design

The design used in this research is qualitative-descriptive as proposed by Ferrini-Mundy and Graham (1994). Because the data of the research are based on the results of interviews, the results of the data collection were analyzed through a qualitative method (Mertler, 2009; Shank, 2002). According to Mertler (2009), qualitative research entails any research methodology that requires the collection and analysis of descriptive data and uses an inductive approach to reasoning.

4 Results, Analyses, and Interpretations

4.1 Results

4.1.1 Misconceptions

Eight misconceptions about the first derivative of a function defined graphically were gathered.

[22] The interviewer gave each interviewee a copy of the test and asked him/her to perform the following steps:

- Read the required Question.
- Expect the way(s) his/her students deal with the question.
- Expect the errors his/her students may do and explain, if possible, why they did each error.
- Tell whether s/he talked about the subject matter in the class or not; if not, explain why.
- Tell which textbook(s) is(are) used in the class

Table 1 summarizes the eight misconceptions and shows the mathematical meaning of each.

Table 1
Misconceptions and Their Mathematical Meaning

Misconceptions	Mathematical Meaning
(1) Function Is a Constant	If f (a) = k, then f'(a) = 0
(2) Algebraic Form	To find f'(a), find f (x) first
(3) Function and Its Derivative Are the Same	f'(a) = f (a)
(4) Relative Extremum	f admits a relative extremum at a even when $f'(a) \neq 0$ OR f does not change its sense of variations at a
(5) Point of Inflection	If $A(x_A, f(x_A))$ is a point of inflection of the representative graph of the function f, then $f'(x_A) = 0$
(6) Discontinuity	If A is a cusp (angular point) in the representative graph of a function f, then f is discontinuous at A
(7) Asymptote	$f'(a) = 0$ if the representative graph of f admits $x = a$ as a vertical asymptote
(8) Non-Differentiability	If f'(a) does not exist, then the representative graph of f admits $x = a$ as a vertical asymptote

4.1.2 Causes

Each of the above misconceptions has its own causes.
Table 2 summarizes the causes of the eight gathered misconceptions deducted from the interviews with coherence of the literature review.

Table 2
Causes of the Eight Gathered Misconceptions

Misconceptions	Causes
(1) Function Is a Constant	The function was defined graphically. Students do not know that a function is a set of points and that the derivative of a function is a function in itself. The teaching instructional method used in introducing the concept.
(2) Algebraic Form	The predominant use of functions given by algebraic formulas in the class.

(3) Function and Its Derivative Are the Same	The lack of discrimination of concepts which occur in the same context. The confusion of a concept with another concept describing a different feature of the same situation. The lack of understanding of graphical representation. The routine questions that include graphical materials related to a function and give less effort on the graphical interpretation of its first derivative. The teaching instructional method. The analogies that students usually make between the rules or formulas they know, without paying attention to all the conditions of the original rule or formula
(4) Relative Extremum	The routine questions that are given in the class and the exams. The teaching instructional methods used in the class. The quantity and quality of exercises and problems solved in the class.
(5) Point of Inflection	The teaching instructional method used in the class because the teacher talked about the concept without going deep in it.
(6) Discontinuity	The lack of knowledge of the teacher about the curriculum.
(7) Asymptote	The confusion of a concept with another concept describing a different feature of the same situation.
(8) Non-Differentiability	The students' graphing skills are weak when cognitively combining, simultaneously, interval information of the derivative and critical points.

4.2 Analyses and Interpretations of the Collected Misconceptions

In this section, each misconception will be explained explicitly with its scientific environment and its eventual cause(s). In addition, some examples will be presented to show how the student expresses it in both the test and the interview[23].

Function Is a Constant

The Lebanese official mathematics textbook for grade 10 defines the function as: "A function from a set D to $\mathbb{R}$ is a rule that assigns a single element of $\mathbb{R}$ to each element in D." It also states that "a function is a mapping from D onto $\mathbb{R}$". The same textbook defines the graph of a function

[23] We don't give the frequency of the misconception because it's not the aim of this paper. If needed, read the thesis.

f as: “Let *f* be a function defined on a domain *D*. The set of points (x, y) where $x \in D$ and $y = f(x)$ is called the graph of *f*.”[24] The above definition states that the value of a function *f* at a point *a* is $y = f(a)$.

We may say that the participants do not know that the derivative of a function is a function in itself; and to find the value of the derivative function at a point, one should find the derivative function first, and then deal with the derivative as a new function.

The graphical representation of the function may be the reason of this misconception among these participants. Because participants do not have the algebraic form of the function, they went on searching for this form by finding any form of $b = f(a)$ rather than $y = f(x)$. In this case, they do not even know that the graph of a function is a set of points as the definition of the graph of a function states. For example:

S78[25]:	(Question 2)	$f'(-2) = 0$ since f(x) is a constant
S76:	(Question 3)	$f(1) = 1$, but we can realize that f(x) is constant at f(1) $\Rightarrow$ when f(x) = constant $\neq 0$, then $f'(x) = 0$ since derivative of a constant = 0

From the interviews with student 78 and his teacher, we could see that the teacher knows what the students might answer correctly, but he does not know what their misconceptions in the subject matter could be. The teaching instructional method used in introducing the concept of number derivative also affect on the students’ way of thinking. Maybe students will do better in the concept if the teacher introduces the number derivative using more than one representation (algebraically, graphically, and table). Schoenfeld *et al.* (1990) found that what appears to most people working at any level in mathematics to be a very simple concept can present serious difficulties.

Algebraic Form

Asiala *et al.* (1997) found that students have a predominant reliance on the need for algebraic formulas when dealing with the function concept.

[24] “Building up Mathematics”, Secondary Education, First Year, pp. 195-196.
[25] “S78” stands for student 78 in the population for the research.

Such a misconception might appear because of the teaching methods that are used in classrooms. Teachers usually, in the concept of functions and/or differentiation, give a short time for the graphical representation; they give more attention to the algebraic form of the function.

Leinhardt *et al.* (1990) suggest that the predominant use of functions given by algebraic formulas may contribute to the development of a restricted notion of function.

In questions 2, 3, 5, 6, 7, and 8, student 5, for example, stated that it is not possible to calculate $f'(a)$ because the algebraic form of function f is not given.

In the interview with student 5, we could see that the participant knows how to deal with the graph of a function graphically, but she doesn't know how to deal with the derivative of a function that is defined graphically. She cannot correlate between the graph of a function and the number derivative of that function.

In an interview with her Teacher, we could see that she is aware of the problem of the textbook used in the school. She also introduces the number derivative graphically, which helps in understanding the concept more deeply.

The Function and Its Derivative Are the Same

Teachers usually concentrate more on routine questions that include graphical materials related to a function, and give less effort on the graphical interpretation of its first derivative. That's why some students, when given a graph, think about the function before they think about its derivative. They begin solving such questions without giving any attention to the derivative symbol. Even those teachers who give graphical materials related to first derivative, they also concentrate on routine questions that are related, as one teacher stated, to relative extremum, horizontal tangent, and rate of change. Thus, having routine and non-routine questions in the class may help in skipping such a misconception.

Orton (1983), in a research about differentiation, confirmed this misconception when he concluded that students confuse the derivative of a function at a point and the value of the function at that point, or the y-value of the point of tangency.

This misconception is multifaceted. It appeared in the research in four different types of errors, and more than one time with some participants.

E_1: $f'(a) = f(a)$

The first error is when some participants found $f'(a)$ by just finding the ordinate of the point with abscissa a, without taking care of the difference between the function f and its derivative function f'. For example:

S33: (Question 6) $f'(3) = -3$ (graphically) where it is the minimum point

(The student marked the point $(3, -3)$ on her graph and showed the abscissa and ordinate of this point by drawing parallel segments to both axes)

The participant confused between the two concepts in the same situation: The value of a function at a point and the value of the derivative of that function at that point. This may happen because of the reason that was stated by Ubuz (2001), who said that:

> Students seem to think about different concepts as the same because of the lack of discrimination of concepts which occur in the same context, the confusion of a concept with another concept describing a different feature of the same situation, or the lack of understanding of graphical representation. (p. 129).

E_2: $f'(a) = \lim_{x \to a} f(x)$

The second error appears in Question 5. Some participants considered that $f'(a)$ is equal to the limit of $f(x)$ as x approaches a, which shows that they do not distinguish between the function f and its derivative function f' and, consequently, they do not know the difference between the continuity and the differentiability of a function at a number. For example:

S3: (Question 5) $f'(-2) = +\infty$ because $\lim_{x \to -2} f(x) = +\infty$

E_3: $a < b$ and f is increasing, then $f'(a) < f'(b)$

The function at a point and its derivative at that point are equal appears, in a third context, when some participants used f' instead of f in the following property: "Let f be a continuous function that is defined on an interval $[a, b]$. If $a < b$ and f is strictly increasing on $[a, b]$, then $f(a) <$

$f(b)$". This error also shows that participants do not know the difference between the function f and its derivative function f'. For example:

S16: (Question 7) $a < b$, then $f'(a) > f'(b)$; $-2 < 1$ and f is increasing, then $f'(-2) > f'(1)$

S31: (Question 8) $1 > -3$; $f'(b) > f'(a)$, then $f'(1) > f'(-3)$

In this case, the teaching instructional method plays a major role in this misconception. The way the teacher introduces a new concept is important. Some students used wrong methods to justify their responses because of the method the teacher used in introducing the concept.
Another aspect of this misconception is that students have the tendency to make analogies between the rules or formulas they know, without taking into consideration all the conditions of the original rule or formula. The error participants did in this context is that they either could not distinguish between the function f and its derivative function f' or they did wrong analogies between the rules they know.

***E_4*: $f'(a) = b$, then the graph of f passes through (a, b)**

The fourth error holds in questions 9 to 12. While drawing the graph, some participants drew graphs that pass through the point (a, b) because they were asked to satisfy the condition $f'(a) = b$ in their graphs. This also confirmed that they do not know how to differentiate between the function f and its derivative function f'. For example:

S52: (Question 10) (The student drew six graphs of six functions all passing through the point $(-2, 0)$)

In an interview with some participants, we could see that the participants know how to deal with the graph of a function graphically, but they do not know how to deal with the derivative of a function that is defined graphically. They cannot correlate between the graph of a function and the number derivative of that function.

Relative Extremum

The Lebanese official mathematics textbook for grade 11 – Sciences section[26] defines the relative extrema as:

[26] "Building up Mathematics", Secondary Education, Second Year, Sciences Section, p. 304.

- Consider a differentiable function f on an open interval I. If f' becomes zero at a while changing signs from negative to positive, then $f(a)$ is a local minimum of f.
- Consider a differentiable function f on an open interval I. If f' becomes zero at b while changing signs from positive to negative, then $f(b)$ is a local maximum of f.

Starting from the above definitions, we see that the participants have the misconception when they concluded a result from a definition without applying all of its conditions. This misconception appeared in questions 3 and 6.

***f* has a relative extremum when *f* '(x) = 0**

In this subsection, the participants responded that the function f has a relative extremum although it does not change its sense of variations or $f'(x)$ does not change its sign. This happened in Question 3. Example:

S89: (Question 3) $f'(1) = 0$ because it is a local maximum

From the interviews conducted, it was clear that the participants do not know that a function has a relative extremum at a if it verifies three conditions: Continuity at a, the derivative is zero at a, and the derivative function changes signs at a. They specified, first, only one condition. With a little prompt from the interviewer, they could specify one more condition, but they could not get the third one. Such students do not, in fact, understand the concept but they are just imitating a similar situation they faced before.

From the interviews with students and their teachers, we see that the causes of the misconception are the routine questions that are given in the class and the exams, the teaching instructional methods used in the class, and the quantity and quality of exercises and problems solved in the class.

A teacher who talks independently about the continuity and differentiability of a function, that is represented algebraically (piecewise) or graphically, does not help his/her students to find the linkage between the two representations. Indeed, such a teacher leads the student to think about the context of the chapter rather than conceptualizing that concept.

Moreover, when students solve a small number of exercises about a certain concept, they rely on the memorized procedures (Heid, 1988). Students usually concentrate more on questions that are frequently solved in the class because they believe that they are typical ones. A teacher who solves only one question about a certain concept and does not ask about it in the exam, as stated one of the teachers, strengthens the reliance of the students on the memorized procedures. Such a teacher ignores the impact of the exam on the students' knowledge.

***f* has a relative extremum at *a* although *f* '(*a*) does not exist**

In this subsection, the participants responded that the function f has a relative extremum at a although the function is not differentiable at a, or $f'(a)$ does not exist. This happened in Question 6: For example:

S24: (Question 6) $f'(3) = 0$ since at $x = 3 \rightarrow y = -3$ which is local minimum. Then $f'(3) = 0$

S67: (Question 6) $x = 3$, $f(3)$ is minimum $\Rightarrow f'(3) = 0$

From the interviews, we could see that the participants could specify only two conditions, out of three, after a little prompt from the interviewer.

From the interviews with their teachers, it was clear that the reason of the misconception was the routine questions that are used in the class. Many teachers complain from the shortage of time in their classes. Maybe the students will benefit if the teacher asks them, when they confront new situations, to go back to the definition and the properties of any concept.

Point of Inflection

The derivative of a function at the point of inflection is null. This is the misconception of the students.

The Lebanese curriculum did not mention the concavity and/or point of inflection in grade 11, but the teachers in our sample have evoked it in their classes. The curriculum mentions, in grade 12, that the student should be able to "calculate the second derivative of a function at a point and on an interval" and "use the second derivative to determine the point of inflection of a curve and show the relation that exists between the sign of the second derivative of a function and the concavity of the representative curve of this function."[27]

Here are two extracts of the participants' responses:

[27] Lebanese Mathematics Curriculum, CRDP, pp 124- 125

S27: (Question 3) $f'(1) = 0$ (point of inflection)

S65: (Question 3) $f'(1) = 0$ (center … concavity)

From the interviews conducted, it seems that students have a misconception because the teacher talked about a concept without going deep in it. Teachers' educational beliefs have their impacts on their students; a teacher who does not give enough time to a certain non-required concept may affect negatively on his/her students and may lead them to have some misconceptions.

Discontinuity

Here are two extracts of the participants' responses:

S83: (Question 6) At $f'(3)$ there is no continuous, so there is no derivative to the function f

S86: (Question 6) No derivative since no continuous

In this case, we see that students know that a discontinuous function is not differentiable, but he does not know how to distinguish between a continuous and discontinuous function graphically. If we look at the Lebanese curriculum, we see that it encourages the graphical approach of the notion of continuity: "The notions of left and right continuity are not part of this program. Since it is difficult to understand the notion of continuity analytically, it will be only approached graphically this year"[28].

From the interview with the teachers, we see that the way the teachers approaches a concept may affect on the understanding of their student. Maybe reversing the way of teaching will make a difference on the understanding of the students. The lack of knowledge of the teacher about the curriculum might also make a difference on the understanding of the students. A teacher who knows the limits and different approaches of a certain concept will help his/her students in understanding this concept better.

Asymptote

Participants responded that $f'(-2) = 0$ since the line of equation $x = -2$ is a vertical asymptote. For example:

[28] Lebanese Mathematics Curriculum, CRDP, p 121

S18: (Question 5) $f'(-2) = 0$

S36: (Question 5) $f'(-2) = 0$ vertical asymptote

From an interview with S18, we could say that the participant knows the analytical definition of the derivative of a function at a point, but she has a problem in calculating the limit of a certain ratio. She could not check whether there is an indeterminate form in the limit she is calculating or not. She deduced that the limit of the ratio is zero because the limit of the numerator is zero. This may happen when the student uses the substitution method in calculating the limit of a discontinuous function. Students think that the substitution method is a method that is applicable on every function. Tall (1996) stated that students have a conflict in using the substitution method to calculate the limit of $2x + h$ as h approaches zero and the limit of the ratio $\frac{(x+h)^2 - x^2}{h}$ as h approaches zero.

Non-Differentiability

In Question 12, participants drew representative graphs of the function f having a vertical asymptote of equation $x = -1$ because $f'(-1)$ does not exist.

From the interview with one of the teachers, she claimed that she ignored one of the conditions because this is not something typical to her, and this maybe a valid reason. She also stated that she hesitated in drawing the correct graph because her first impression about non-differentiability of a function at a point is discontinuity. The claims of the teacher explain the misconception of the participants. The participants could not make a link between all the conditions of the question at the same time, especially what is related to the last one, that's why they ignored it. Moreover, the participants may know that the derivative of a function at a point does not exist means that the function is not differentiable, and that a non-differentiable function could be continuous or not, but they could not tell that, in this case, the graph should have a cusp. Baker *et al.* (2000) talked about this point and found that students' graphing skills are weak when cognitively combining, simultaneously, interval information of the derivative and critical points. In addition, the teaching and evaluation methods that are used in the class may explain the misconception of the students. The teacher said that she usually gives typical questions in the class and does not go to this extent in the concept. Maybe teaching a concept from different perspectives helps students deal with new situations easier.Because typical undergraduate courses in calculus often rely on the concept of derivative to explain the differentiability of functions and to define integrals and differential equations, a student holding misconceptions of the derivative concept has the risk of developing flawed conceptions of these later topics that will negatively impact the rest of his/her mathematical understanding.

5 Conclusions

The primary purpose of this paper is to highlight the misconceptions of grade 11 students and their possible causes. This will enable teachers and also trainers and teachers of mathematics' teachers to be aware of them and take them into consideration in their practices[29].

The test and the interviews showed the following misconceptions:

(1) The value of a function at a point is a constant, so its derivative is zero.
(2) The derivative of a function at a point cannot be calculated unless its algebraic (explicit) form is given or found.
(3) The value of a function at a point and the value of its derivative at that point are equal.
(4) A function has a relative extremum at a point *a* even if its derivative at *a* is not zero or it does not change its sense of variations.
(5) The derivative of a function at the point of inflection is null.
(6) The derivative of a function at a cusp (angular point) does not exist because the function is not continuous at that point.
(7) The derivative of a function at a point is zero when the representative graph of that function admits a vertical asymptote at that point.
(8) The representative graph of a function admits the vertical asymptote $x = a$ if $f'(a)$ does not exist.

The causes of these misconceptions vary among three major axes:

(1) <u>The student</u>: Some of the misconceptions are made because students make analogies among the formulas they know without taking into consideration the specific conditions each formula has.

[29] For more details, read the thesis mentioned before.

Some of the misconceptions also happen because students are weak in the graphing skills, especially when they have more than one piece of information about the derivative concept at the same time (as Baker *et al.* (2000) found).

(2) The teacher: The teaching instructional method used in introducing the concept of derivative, concentrating more on routine questions related to algebraic form of the function rather than giving more effort on the graphical interpretation of the derivative, play a role in having these misconceptions (as Leinhardt *et al.* (1990) and Selden *et al.* (1994) found).
In addition, the knowledge of the teacher about the curriculum, and the quantity and quality of exercises and problems solved in the class help in having these misconceptions.

(3) The concept: The routine aspects of differentiation, the lack of discrimination of concepts which occur in the same context, the confusion of a concept with another concept describing a different feature of the same situation, and the lack of understanding of graphical representation contribute in creating these misconceptions (as Orton (1983) and Ubuz (2001) found).

According to the results, teachers are recommended to giving routine and non-routine questions in the classroom and the exams and to giving examples and counter-examples. They are also recommended to working out (a) the difference(s) between a function and its derivative at a point; (b) the graphical difference(s) between a null derivative at a point and a non-null one; (c) the relation between the slope of a tangent line at a given point on a graph of a function and the derivative of that function at the abscissa of that point; (d) the existence of the derivative, or not, of a function at a given point, in different situations; (e) and constructing a graph having one or more conditions dealing with a function and its derivative at a given point(s).

Obviously, these recommendations are to be evaluated through future researches.

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Lebanese Mathematics Textbooks

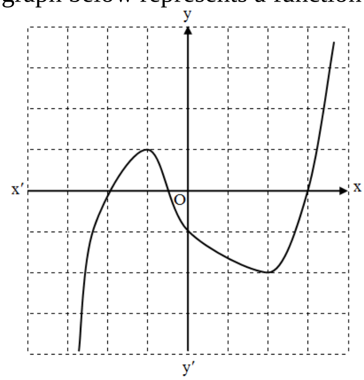
"Building up Mathematics", Secondary Education, First Year.

"Building up Mathematics", Secondary Education, Second Year, Sciences Section.

APPENDIX A

Question 1

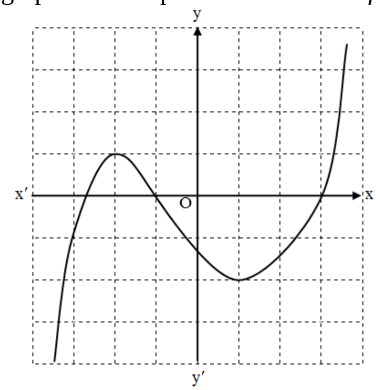
The graph below represents a function *f*.



Study the sign of $f'(x)$. Explain your work.

Question 2

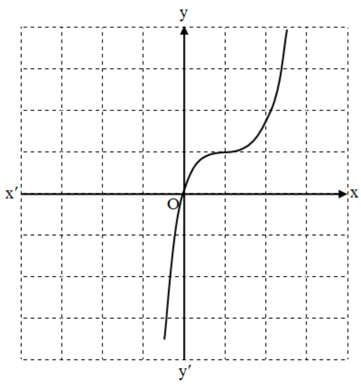
The graph below represents a function *f*.



Find, if possible, $f'(-2)$. Explain your work.

Question 3

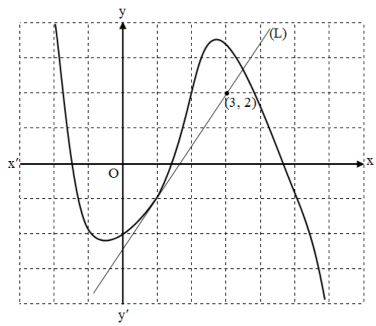
The graph below represents a function *f*.



Find, if possible, $f'(1)$. Explain your work.

Question 4

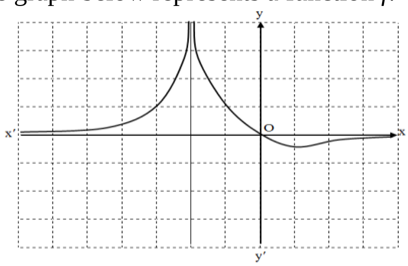
The graph below represents a function *f*.
(L) is tangent to the graph at point $(1, -1)$.



Find, if possible, $f'(1)$. Explain your work.

Question 5

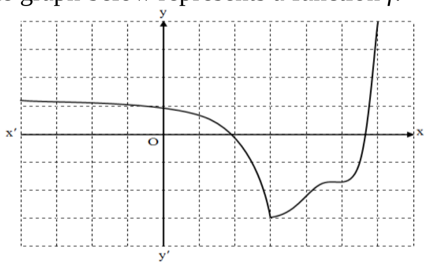
The graph below represents a function *f*.



Find, if possible, $f'(-2)$. Explain your work.

Question 6

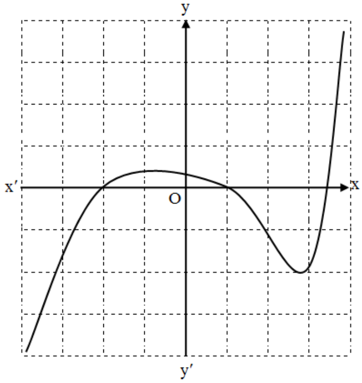
The graph below represents a function *f*.



Find, if possible, $f'(3)$. Explain your work.

Question 7

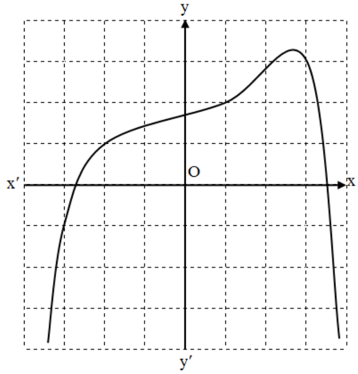
The graph below represents a function *f*.



Compare $f'(-2)$ and $f'(1)$. Explain your work.

Question 8

The graph below represents a function *f*.



Compare $f'(-3)$ and $f'(1)$. Explain your work.

Question 9

Draw a graph of a function *f*, knowing that $f'(2) = 0$.

Question 10

Draw a graph of a function *f*, knowing that $f'(-2) = 0$ and f is a decreasing function.

Question 11

Draw a graph of a function *f*, knowing that $f'(2) = 3$.

Question 12

Draw a graph of a function *f*, knowing that

- *f* is continuous on $]-\infty\ , 1[$ and on $]1\ , +\infty\ [$
- $\lim\limits_{x \to 1^-} f(x) = +\infty$; $\lim\limits_{x \to 1^+} f(x) = -\infty$
- $f'(-3) = f'(-2) = f'(2) = 0$
- $f'(x) > 0$ when $x \in\ ]-2\ , 1[$ and when $x \in\ ]1\ , 2[$
- $f'(x) < 0$ when $x \in\ ]-\infty\ , -3[$ and when $x \in\ ]-3\ , -2[$ and when $x \in\ ]2\ , +\infty\ [$
- $f'(-1)$ does not exist